Harvard University
Computer Science 121

Problem Set 5

Due Tuesday, October 22, 2013 at 1:20 PM.
Submit your solutions electronically to cs121+ps5@seas.harvard.edu with "ps5 submission" in the subject line. The solutions to Parts A and B should be attached as separate PDF files, called lastname+ps5a.pdf and lastname+ps5b.pdf.
Late problem sets may be turned in until Friday, October 25, 2013 at 1:20 PM with a 20% penalty.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

PART C (Graded by Nick)

PROBLEM 1 (15 points)

Let ROMANIAN denote the set of all grammatical sentences in Romanian over an alphabet Σ consisting of all words in Romanian. Let S denote the set of all Romanian sentenes as defined below:

$$S = \{(\text{Pe cine})^i\ (\text{cui})^j\ \text{ai vrut}\ (\text{sa rogi})^i\ (\text{sa spuna})^j\ \text{povestea:}\ i, j > 0\}$$

Let $f : \{a, b, c, d, x, y\}^* \to \Sigma^*$ be the following homomorphism:

$$f(w) = \begin{cases} \text{pe cine} & \text{if } w = a \\ \text{cui} & \text{if } w = b \\ \text{sa rogi} & \text{if } w = c \\ \text{sa spuna} & \text{if } w = d \\ \text{ai vrut} & \text{if } w = x \\ \text{povestea} & \text{if } w = y \end{cases}$$

(A) Given that a string "$(\text{Pe cine})^{i_0}\ (\text{cui})^{j_0}\ \text{ai vrut}\ (\text{sa rogi})^{i_1}\ (\text{sa spuna})^{j_1}\ \text{povestea}" \in \Sigma^*$ is in ROMANIAN if and only if $i_0 = i_1 > 0$ and $j_0 = j_1 > 0$, describe the following language L in set notation:

$$L = f^{-1}(\text{ROMANIAN} \cap ((\text{pe cine})^*(\text{cui})^*\ \text{ai vrut}\ (\text{sa rogi})^*(\text{sa spuna})^*\ \text{povestea}))$$

(B) Using L, argue that ROMANIAN is non-context-free.

(Note: For those interested, we give the translation of the sentences of S below. The top line is the Romanian, the middle line is a word for word translation, and the bottom line is the English translation.)

(1) (Pe cine)^i (cui)^j ai vrut (sa rogi)^i (sa spuna)^j povestea?
(who.ACC)^i (who.DAT)^j have wanted (to ask)^i (to tell)^j story?
"Who have you wanted to ask to ask who ... to tell who... to tell who the story?"

PROBLEM 2 (5+5 points)

Let $G = (V, \Sigma, R, S)$ where $V = \{S, V\}$, $\Sigma = \{a, b\}$, and R is the set of rules:

$$\begin{aligned} S &\rightarrow bSS \mid aS \mid aV \\ V &\rightarrow aVb \mid bVa \mid VV \mid \varepsilon \end{aligned}$$

(A) Transform G into an equivalent grammar G' in Chomsky normal form.

(B) Verify that the string *abaab* is generated by G', using the recognition algorithm for grammars in Chomsky normal form given in class. Show the complete filled-in matrix.