

Harvard University Extension School
Computer Science E-121

Problem Set 5

Due Friday, October 25, 2013 at 11:59 PM Eastern Time.

Submit your solutions in a single PDF called lastname+ps5.pdf emailed to cscie121@seas.harvard.edu.

LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by ** ENTER YOUR NAME HERE **

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

PROBLEM 1 (5+5 points)

Let $G = (V, \Sigma, R, S)$ where $V = \{S, V\}$, $\Sigma = \{a, b\}$, and R is the set of rules:

$$\begin{aligned} S &\rightarrow bSS \mid aS \mid aV \\ V &\rightarrow aVb \mid bVa \mid VV \mid \varepsilon \end{aligned}$$

(A) Transform G into an equivalent grammar G' in Chomsky normal form.

(B) Verify that the string $abaab$ is generated by G' , using the recognition algorithm for grammars in Chomsky normal form given in class. Show the complete filled-in matrix.

PROBLEM 2 (3+3+3+3+3 points)

True or false? Justify your answers.

- (A) $\{a^{n^2} : n \geq 0\}$ is context free.
- (B) $\{a^n b^m : n < m < 2n\}$ is context free.
- (C) $\{a^n b^* a^n b^* a^n : n \in \mathbb{N}\}$ is context free.
- (D) If L is context-free and R is regular, then $L - R$ is context-free.
- (E) If L is context-free and R is regular, then $R - L$ is context-free.

PROBLEM 3 (5 points)

Prove that if M is a PDA and there exists a number k such that for all $w \in \mathcal{L}(M)$, the size of the stack is at most k in each step of every possible computation of w on M , then $\mathcal{L}(M)$ is regular.

PROBLEM 4 (10 points)

Show that if A is context free and B is regular then $A/B = \{w : wx \in A \text{ for some } x \in B\}$ is context free.

PROBLEM 5 (10 points)

Recall that given a function $f : \Sigma \rightarrow \Delta^*$, the function $h : \Sigma^* \rightarrow \Delta^*$ defined recursively by $h(\epsilon) = \epsilon$ and $h(w\sigma) = h(w)f(\sigma)$ is called a homomorphism.

(A) Show that if L is a context-free language over the alphabet Σ and $h : \Sigma^* \rightarrow \Delta^*$ is a homomorphism, then $h(L) = \{h(w) : w \in L\}$, is a context-free language over the alphabet Δ .

(B) Show that if L is a context-free language over the alphabet Δ and $h : \Sigma^* \rightarrow \Delta^*$ is a homomorphism, then $h^{-1}(L) = \{w : h(w) \in L\}$ is a context-free language over the alphabet Σ .

PROBLEM 6 (15 points)

Let ROMANIAN denote the set of all grammatical sentences in Romanian over an alphabet Σ consisting of all words in Romanian. Let S denote the set of all Romanian sentences as defined below:

$$S = \{(\text{Pe cine})^i (\text{cui})^j \text{ ai vrut } (\text{sa rogi})^i (\text{sa spuna})^j \text{ povestea} : i, j > 0\}$$

Let $f : \{a, b, c, d, x, y\}^* \rightarrow \Sigma^*$ be the following homomorphism:

$$f(w) = \begin{cases} \text{pe cine} & \text{if } w = a \\ \text{cui} & \text{if } w = b \\ \text{sa rogi} & \text{if } w = c \\ \text{sa spuna} & \text{if } w = d \\ \text{ai vrut} & \text{if } w = x \\ \text{povestea} & \text{if } w = y \end{cases}$$

(A) Given that a string “(Pe cine)^{i₀} (cui)^{j₀} ai vrut (sa rogi)^{i₁} (sa spuna)^{j₁} povestea” $\in \Sigma^*$ is in ROMANIAN if and only if $i_0 = i_1 > 0$ and $j_0 = j_1 > 0$, describe the following language L in set notation:

$$L = f^{-1}(\text{ROMANIAN} \cap ((\text{pe cine})^*(\text{cui})^* \text{ ai vrut } (\text{sa rogi})^*(\text{sa spuna})^* \text{ povestea}))$$

(B) Using L , argue that ROMANIAN is non-context-free.

(Note: For those interested, we give the translation of the sentences of S below. The top line is the Romanian, the middle line is a word for word translation, and the bottom line is the English translation.)

- (1) (Pe cine)ⁱ (cui)^j ai vrut (sa rogi)ⁱ (sa spuna)^j povestea?
 (who.ACC)ⁱ (who.DAT)^j have wanted (to ask)ⁱ (to tell)^j story?
 “Who have you wanted to ask to ask who ... to tell who... to tell who the story?”

PROBLEM 7 (Challenge! 3 points)

Write a context-sensitive grammar for the language $\{a^n b^n c^n : n \geq 0\}$.