Harvard University
Computer Science 121
Problem Set 8

Due Tuesday, November 12, 2013 at 11:59 PM.
Submit your solutions electronically to cs121+ps8@seas.harvard.edu with "ps8 submission" in the subject line. The solutions to Parts A and B should be attached as separate PDF files, called lastname+ps8a.pdf and lastname+ps8b.pdf.
Late problem sets may be turned in until Friday, November 15, 2013 at 11:59 PM with a 20% penalty.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

PART A (Graded by Francisco and Anupa)

PROBLEM 1 (4+8+8 points)

You may find it useful to review Rice's theorem and its proof before doing this problem (Lecture 17 Slides 10-11). Let L_0 be any arbitrary but fixed Turing-recognizable language other than $\emptyset$ or Σ^*. We will prove that the language $X = \{\langle M \rangle : L(M) = L_0\}$ is neither Turing-recognizable nor co-recognizable.

(A) Prove that $A_{\overline{\varepsilon}} = \{\langle M \rangle : \varepsilon \notin L(M)\}$ is not Turing-recognizable. (*Hint:* this can be done without a reduction.)

(B) Prove that X is not Turing-recognizable by giving a *mapping* reduction from $A_{\overline{\varepsilon}}$ to X.

(C) Prove that $\overline{X}$ is not Turing-recognizable, again by reduction from $A_{\overline{\varepsilon}}$.

PROBLEM 2 (3+3+3 points)

Let $f(n)$ and $g(n)$ be functions over: positive natural numbers $\rightarrow$ positive real numbers. Prove or give a counterexample for the following conjectures (and explain briefly why any given counterexamples are valid).

(A) $f(n) = \Theta(g(n))$ implies $g(n) = \Theta(f(n))$.

(B) $f(n) = O((f(n))^2)$.

(C) $f(n) = \Theta(g(n))$ implies $2^{f(n)} = \Theta(2^{g(n)})$.

PROBLEM 3 (Challenge! 2 points)

Show that every infinite regular language has a subset that it recognizable but not decidable.