CS 121 Section 7

Harvard University

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1 Overview

This week we are covering universal TMs, TM encodings, nondeterministic TMs, dovetailing simulation, enumerators, TM algorithms, and high-level descriptions of TMs.

1.1 High-level descriptions

Given the ChurchTuring Thesis and representation independence, we no longer need to refer to a specific computing model or or data representation when describing an algorithm. Instead:

- Describe it as a sequence of steps operating on higher-level data types (e.g. numbers, graphs, automata, grammars).
- Each step: simple enough that it is clear it can be implemented on a reasonable model (such as a TM) using a reasonable data representation.
- Freely make use of algorithms we have seen (or are well-known, such as elementary arithmetic) as subroutines.
- Freely make use of control-flow primitives, such as loops, if-then-else, gotos, etc.

1.2 Encodings and Universal TMs

We can encode complex data into strings over a small alphabet. e.g. we denote the encoding of a TM M as $\langle M \rangle$.

On input $\langle M, x \rangle$ a universal TM U simulates M when run on input x. i.e. it accepts/rejects/'loops' if and only if $M(x)$ accepts/rejects/loops.

1.3 Enumerators

A language is enumerable if its elements can be listed by a TM (with the ability to output a list of strings, not just accept/reject).

1.4 NTMs and Dovetailing

A nondeterministic TM (like a PDA or NFA) can follow multiple computation paths. It accepts if any of the computation paths accepts.

NTMs are equivalent to TMs. (Every language recognized by a NTM is recognized by a TM and vice versa. What about decidable languages?) We proved this equivalence by dovetailing: we simulate each possible computation path, but we have to be careful not to follow infinitely long paths.

2 Excercises

Exercise 2.1. *Show that the class of decidable languages is closed under intersection.*

Exercise 2.2. *Let* $L = \{\langle M \rangle \mid M$ *is a DFA and for every string* w*, if* M *accepts* w*, then* M *also accepts* $w^R\}$*. Show that* L *is decidable.*

Exercise 2.3. *Show that a language* L *is decidable if and only if there is an enumerator that outputs the elements of* L *in lexicographic order.*

Exercise 2.4. *Let* $L = \{\langle M \rangle \$x : M$ *only uses the first* $|x|$ *cells of the tape when run on* $x\}$*. Show that* L *is decidable.*