

**Harvard University
Computer Science 121**

Problem Set 0

Due Tuesday, September 10, 2013 at 11:59 PM.

Submit your solutions electronically on the course website, located at <http://people.seas.harvard.edu/~salil/cs121/fall12/>. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A and B, in separate files named lastname+ps0a.pdf and lastname+ps0b.pdf respectively, in the appropriate dropboxes.

See syllabus for collaboration policy.

Note: This is our standard prologue. Problem set 0 counts 0 points, but you should complete it, and we will grade it and provide a solution set for your edification.

PART B (Graded by Nick)

PROBLEM 1 (5+5+5+5 points)

Let L_1 be the language $\{a^n : n \geq 0\}$ and L_2 be the language $\{x : x \in \{a, b\}^* \text{ and } |x| = 5\}$.

(A) Which of the following sets are finite? Of those that are finite, what is their cardinality?

- i. $L_1 \cap L_2$
- ii. $L_2 \times (L_1 \cap L_2)$
- iii. $L_2 \setminus L_1$.

(B) Which of the following sets contain the empty string ε ?

- i. $L_1 \cap L_2$
- ii. $L_1 \cup L_2$

(C) Which of the following sets have the empty set \emptyset as a subset?

- i. L_2
- ii. $L_1 \cap L_2$

(D) Which of the following sets contain \emptyset as an element?

- i. L_1
- ii. $P(L_2)$

PROBLEM 2 (Sipser 0.13 Challenge! 3 points)

An undirected graph is a set of points with lines connecting some of the points. The points are called nodes or vertices, and the lines are called edges. The number of edges at a particular node is the degree of that node. An edge from a node to itself is called a self-loop. (Sipser, pg. 10)

Show that every graph without self loops and with two or more nodes contains two nodes that have equal degrees.