Harvard University Extension School
Computer Science E-121

Problem Set 0

Due Friday, September 13, 2013 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps0.pdf emailed to cscie121@seas.harvard.edu.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.
See syllabus for collaboration policy.

Note: This is our standard prologue. Problem set 0 counts 0 points, but you should complete it, and we will grade it and provide a solution set for your edification.

PROBLEM 1 (2+2+2+2 points)

Using set notation, give formal descriptions of the following sets:
(A) The set containing no elements.
(B) The set containing the empty string.
(C) The power set of a set X, denoted $\mathcal{P}(X)$, i.e., the set containing all subsets of X.
(D) The difference between two sets X and Y, denoted $X \setminus Y$, i.e., the set containing all elements of X that are not elements of Y.

PROBLEM 2 (10+10 points)

Given a set X, we define the power set $\mathcal{P}(X)$ to be the set of all subsets of X.
(A) Construct a bijection between $\mathcal{P}(X)$ and the set of functions from X into the set $\{0, 1\}$.
(B) Prove that for any finite set X, $|P(X)| = 2^{|X|}$. (Hint: use induction on $|X|$.)

(TURN OVER!)

PROBLEM 3 (5+5+5+5 points)

Let L_1 be the language $\{a^n : n \geq 0\}$ and L_2 be the language $\{x : x \in \{a, b\}^* \text{ and } |x| = 5\}$.

(A) Which of the following sets are finite? Of those that are finite, what is their cardinality?

i. $L_1 \cap L_2$

ii. $L_2 \times (L_1 \cap L_2)$

iii. $L_2 \setminus L_1$.

(B) Which of the following sets contain the empty string ε?

i. $L_1 \cap L_2$

ii. $L_1 \cup L_2$

(C) Which of the following sets have the empty set $\emptyset$ as a subset?

i. L_2

ii. $L_1 \cap L_2$

(D) Which of the following sets contain $\emptyset$ as an element?

i. L_1

ii. $P(L_2)$

PROBLEM 4 (Sipser 0.13 Challenge! 3 points)

An undirected graph is a set of points with lines connecting some of the points. The points are called nodes or vertices, and the lines are called edges. The number of edges at a particular node is the degree of that node. An edge from a node to itself is called a self-loop. (Sipser, pg. 10)

Show that every graph without self loops and with two or more nodes contains two nodes that have equal degrees.