

**Harvard University
Computer Science 121**

Problem Set 0

Due Tuesday, September 10, 2013 at 11:59 PM.

Submit your solutions electronically to `cs121+ps0@seas.harvard.edu` with "ps0 submission" in the subject line. The solutions to Parts A and B should be attached as separate PDF files, called `lastname+ps0a.pdf` and `lastname+ps0b.pdf`.

See syllabus for collaboration policy.

Note: This is our standard prologue. Problem set 0 counts 0 points, but you should complete it, and we will grade it and provide a solution set for your edification.

PART A (Graded by Gabe)

PROBLEM 1 (2+2+2+2 points)

Using set notation, give formal descriptions of the following sets:

- (A) The set containing no elements.
- (B) The set containing the empty string.
- (C) The power set of a set X , denoted $\mathcal{P}(X)$, i.e., the set containing all subsets of X .
- (D) The difference between two sets X and Y , denoted $X \setminus Y$, i.e., the set containing all elements of X that are not elements of Y .

PROBLEM 2 (10+10 points)

Given a set X , we define the power set $\mathcal{P}(X)$ to be the set of all subsets of X .

- (A) Construct a bijection between $\mathcal{P}(X)$ and the set of functions from X into the set $\{0, 1\}$.
- (B) Prove that for any finite set X , $|\mathcal{P}(X)| = 2^{|X|}$. (Hint: use induction on $|X|$.)

PART B (Graded by Nick)

PROBLEM 3 (5+5+5+5 points)

Let L_1 be the language $\{a^n : n \geq 0\}$ and L_2 be the language $\{x : x \in \{a, b\}^* \text{ and } |x| = 5\}$.

- (A) Which of the following sets are finite? Of those that are finite, what is their cardinality?

- i. $L_1 \cap L_2$
- ii. $L_2 \times (L_1 \cap L_2)$
- iii. $L_2 \setminus L_1$.

- (B) Which of the following sets contain the empty string ε ?

i. $L_1 \cap L_2$

ii. $L_1 \cup L_2$

(C) Which of the following sets have the empty set \emptyset as a subset?

i. L_2

ii. $L_1 \cap L_2$

(D) Which of the following sets contain \emptyset as an element?

i. L_1

ii. $P(L_2)$

PROBLEM 4 (Sipser 0.13 Challenge! 3 points)

An undirected graph is a set of points with lines connecting some of the points. The points are called nodes or vertices, and the lines are called edges. The number of edges at a particular node is the degree of that node. An edge from a node to itself is called a self-loop. (Sipser, pg. 10)

Show that every graph without self loops and with two or more nodes contains two nodes that have equal degrees.