NAME:

Harvard University
Computer Science 121 — Final Exam, Friday, December 13, 2013

This is a closed-book examination. You may use any result from lecture, Sipser, or the problem sets, as long as you quote it clearly. The alphabet is $\Sigma = \{a, b\}$ except where otherwise stated.

You have 180 minutes. The problems total to 180 points. Write your name above and on all bluebooks you use. Do the first problem on this page and turn it in along with your blue books. Good luck!

PROBLEM 1 (19 points)

DO THIS PROBLEM ON THIS PAGE!

Complete the following table with `YES`, `NO`, or `?? (CURRENTLY UNKNOWN)`. No explanations needed. M stands for a Turing machine, G_1 and G_2 are context-free grammars, and $\Sigma = \{a, b\}$.

Language:	regular	CF	recursive	r.e.	co-r.e.	$\mathcal{P}$	$\mathcal{NP}$	co-$\mathcal{NP}$
$\{a^n b^{2n} a^{3n} : n \geq 0\}$								
$\{a^n : n = 2^k$ for some $k \geq 0\}$								
$\overline{\text{3-SAT}}$ (the complement of 3-SAT)								
$\{\langle M \rangle : M$ accepts $abab\}$								
$\{\langle G_1, G_2 \rangle : L(G_1) \cap L(G_2) = \emptyset\}$								

PROBLEM 2 (10 points)

Draw a DFA that accepts a string if and only if every occurrence of a is immediately followed by at least two consecutive b's.

PROBLEM 3 (32 points)

(a) Prove that any n-state DFA that accepts at least one string accepts a string of length $< n$.

(b) Prove that any n-state NFA that accepts at least one string accepts a string of length $< 2^n$.

(c) Prove that the set of Turing machines that accept at least one string is recognizable (r.e.).

(d) Prove that the set of Turing machines that accept at least one string is undecidable.

PROBLEM 4 (18 points)

Write regular expressions for these languages or explain why it can't be done.

(a) All strings of the form $a^i b^j$ where i is a multiple of 3 and j is a multiple of 5.

(b) All strings without consecutive b's.

PLEASE TURN OVER

PROBLEM 5 (21 points)

Which of the following are necessarily true for any languages A and B? True/False only, no explanations needed.

(a) If A is cofinite then A is regular.

(b) If A is regular then A has countably many regular subsets.

(c) If A is not recursive then A has countably many non-recursive subsets.

(d) $(A \cup B) - B = A$

(e) A^* is nonempty.

(f) A^* is infinite.

(g) $(A^*)^* = A^*$

PROBLEM 6 (20 points)

(a) Write a context-free grammar that generates all regular expressions over $\{a, b\}$. (It's OK if it generates redundant parentheses.)

(b) For any language L, let $S(L)$ (for "sandwich") be the language $\{xyx^R : y \in L, x \in \Sigma^*\}$. Show that the class of context-free languages is closed under S.

(c) Show that the class of regular languages is not closed under S.

(d) Find a regular language such that $S(L)$ is regular.

PROBLEM 7 (30 points)

(a) Define NP-complete.

(b) Draw a diagram illustrating the likely relations between P, NP, co-NP, and the NP-complete, recursive, r.e., and co-r.e. sets on the hypothesis that P$\neq$NP.

(c) Can an NP-complete set be polynomial-time reducible to a proper subset of itself? Explain.

PROBLEM 8 (30 points)

(a) Let SatFive be the set of Boolean formulas that are satisfied by a truth-assignment in which at most 5 of the Boolean variables are true. Explain why SatFive $\in$ P.

(b) Explain why, if P $=$ NP, it would be possible not only to determine in polynomial time whether a formula is satisfiable but, if so, to find a satisfying truth-assignment.

(c) Let Independent Set be the set of all $\langle G, k\rangle$ for which undirected graph G has a set S of k vertices such that no pair of vertices in S is connected by an edge in G. Prove that Independent Set is NP-complete. (Hint: Reduce from Clique.)

THE END