Name: ______________________ TF: ______________________

Harvard University Extension School
Computer Science E-207
Professor Harry Lewis

Final Examination
December 20, 2011

You should answer the Problem 1 directly on this sheet. Please write your name on the exam and return it with the blue book. Do the rest of the problems in the blue book. The points total 100.

$\Sigma = \{a, b\}$, D is a DFA and M is a Turing machine throughout. QBE is the set of all quantified boolean expressions, for example $\forall x \exists y \forall z[(x \wedge y) \vee (\neg x \wedge \neg z)]$.

PROBLEM 1 (25 points)

Fill the blank entries of the following table with `YES`, `NO`, or `??` ("currently unknown"). No explanations needed.

Language:	finite	regular	context-free	recursive	r.e.	$\mathcal{P}$	$\mathcal{NP}$
$\{ww^{\mathrm{R}} : w \in \Sigma^*\}^*$							
$L(a^*b^*) \cap L(b^*a^*)$							
$\{\langle D_1, D_2\rangle : L(D_1) \cup L(D_2) = \Sigma^*\}$		X	X				
True members of QBE			X				

PROBLEM 2 (32 points)

Answer each question briefly, by arguing the positive or giving a counterexample.
(A) The complement of a context-free language is necessarily recursive.
(B) It is possible to determine algorithmically whether a DFA accepts only finitely many strings.
(C) It is currently unknown whether every $\mathcal{NP}$ language is recursive.
(D) The set difference between two languages in $\mathcal{P}$ is also in $\mathcal{P}$.

PROBLEM 3 (10 points)

Draw a DFA that accepts a string if and only if each occurrence of the substring aa is followed immediately by the substring bb.

PROBLEM 4 (10 points)

Draw a diagram showing the relations among these language classes: recursive, r.e., co-r.e., $\mathcal{P}$, $\mathcal{NP}$, $\mathcal{NP}$-complete, co-$\mathcal{NP}$,
(A) on the assumption that $\mathcal{P} = \mathcal{NP}$;
(B) on the assumption that $\mathcal{P} \neq \mathcal{NP}$

(TURN OVER!)

PROBLEM 5 (10 points)

Show that $\{\langle M, w, 1^k \rangle : M$ halts on input w in at most k time steps$\}$ is in $\mathcal{P}$.

PROBLEM 6 (10 points)

(A) Fill in the ellipsis (...), defining any terms other than "graph," "node," and "edge":

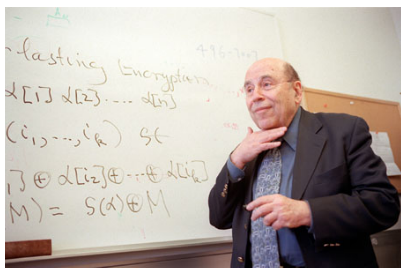
$$\textsc{Vertex Cover} = \{\langle G, k \rangle : \ldots\}$$

(B) We proved in class that Vertex Cover is $\mathcal{NP}$-complete. Does it matter whether k is represented in unary or binary? Why or why not?

PROBLEM 7 (3 points)

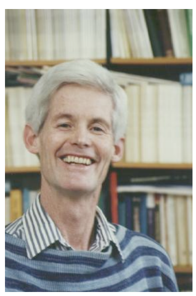
Name these CS121 heroes:

(A) 

(B) 

(C) 

(D) 

(E) 

(F) 

THE END