Harvard University
Computer Science 121
Midterm — October 23, 2012

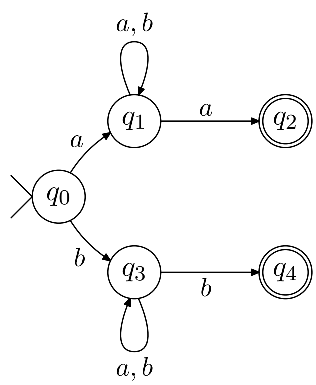
This is a closed-book examination. You may use any result from lecture, Sipser, problem sets, or section, as long as you quote it clearly. The alphabet is $\Sigma = \{a, b\}$ except where otherwise stated. You have 80 minutes. The problems total 75 points. Use pen and write your name on all bluebooks you use. Good luck!

PROBLEM 1 (1+1+1+1+1 points)

For each of the following, say whether it is a string, language, or neither. No justification necessary.
(A) Σ^* (B) ε (C) $\emptyset$ (D) $\{\emptyset\}$ (E) $\{\varepsilon\}$

PROBLEM 2 (3+4+3+5 points)

Consider the following NFA N:



(A) Which of the following strings are accepted by N? *bb*, *abaa*, *abb*.
(B) Write out the formal 5-tuple for N.
(C) Describe in English the language $L(N)$.
(D) Convert the NFA to a DFA using the Subset Construction. (You may simply draw the state diagram of the DFA, omitting unreachable states.)

PROBLEM 3 (5 points)

For a string $w \in \{a, b\}^*$, define $\overline{w}$ to be the string where all a's are replaced with b's and vice-versa. For example, $\overline{aabab} = bbaba$. Give a context-free grammar for the language $\{w \in \{a, b\}^* : w^R = \overline{w}\}$.

PROBLEM 4 (4+4+4+4 points)

TRUE or FALSE? Justify your answers in one or two sentences.
(A) If L_1 is regular and $L_2 \subseteq L_1$, then L_2 is regular
(B) $bababbba \in L(((aba \cup ba)^*b)^*)$
(C) There exists a language L such that L^* is uncountable
(D) The grammar $S \to SS|a$ is ambiguous

PROBLEM 5 (6+6+6+6 points)

For each of the following languages, say whether it is regular, context-free, both, or neither. Briefly justify your answers.
(A) $L = \{a^i b^j : i - j = 2012\}$
(B) $L = \{a^i b^j : i + j = 2012\}$
(C) $L = \{a^n b^n a^n : n \geq 0\}$
(D) $L = \{w : w$ contains both *obama* and *romney* as substrings$\}$, over alphabet $\Sigma = \{a, b, \ldots, z\}$

PROBLEM 6 (10 points)

For a string $w \in \{a, b\}^*$, define odd(w) to be the string consisting of the symbols in odd-numbered positions in w. That is, if $w = w_1 w_2 w_3 \cdots w_n$, then odd$(w) = w_1 w_3 w_5 \cdots w_m$, where m is either n or $n - 1$ depending on whether n is odd or even.

Show that if L is regular, then so is $\{w \in \{a, b\}^* : \text{odd}(w) \in L\}$.

PROBLEM 7 (CHALLENGE! 1 points)

Prove that if R is a regular expression, then $L(R)$ satisfies the pumping lemma with a pumping length equal to $|R|+1$, where $|R|$ is the size (or length) of R. (You can omit the $|xy| \leq p$ condition.)

THE END
REMEMBER TO PUT YOUR NAME ON YOUR WORK.