

Harvard University
Computer Science 121

Quiz — October 28, 2008

You have 80 minutes to complete the quiz. Points total 80. No notes, books, or other aids are allowed. You may use any result already proved in a lecture or on a problem set.

$\Sigma = \{a, b\}$ unless otherwise specified.

PROBLEM 1 (2+2+2+2 points)

Which of the following strings are generated by the regular expression $(ab\varepsilon)^*(a \cup b \cup \emptyset)ba$? (Answer YES or NO; no explanation needed.)

- (A) ε (B) aba (C) $ababba$ (D) $abababa$

PROBLEM 2 (8 points)

Let G be the context-free grammar given by the rules

$$\begin{aligned} S &\rightarrow aSb|Y \\ Y &\rightarrow bY|Ya|\varepsilon \end{aligned}$$

Draw the state diagram of a PDA that recognizes $L(G)$.

PROBLEM 3 (4+4+4+4 points)

True or False? Write a sentence or two explaining your answer.

- (A) Every language is countable.
(B) If $L_1 \cap L_2$ is regular, then L_1 and L_2 are regular.
(C) If L is non-regular, then so is the complement of L .
(D) If $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ are DFAs such that $Q_1 = Q_2$, $s_1 = s_2$, and $F_1 = F_2$, then $L(M_1) = L(M_2)$.

(TURN OVER!)

PROBLEM 4 (7+7+7 points)

For each of the following languages, determine whether it is regular and whether it is context-free.

Justify your answers.

(A) $\{a^n(bc)^n : n \geq 0\}$

(B) $\{a^{2^n} : n \geq 0\}$

(C) $\{a^n a^n a^n : n \geq 0\}$

PROBLEM 5 (9+9 points)

(A) Outline a general procedure for converting a regular expression R into a regular expression R' such that $L(R')$ is the complement of $L(R)$. Be sure to state the type (DFA, RE, etc.) of each intermediate object constructed in the process, and explain informally (in a sentence or two) how each object is obtained from the previous one.

(B) Apply your algorithm (showing all steps) to the regular expression $R = \varepsilon$.

PROBLEM 6 (9 points)

For any language $L \subseteq \Sigma^*$, define $SUFFIX(L) = \{v \in \Sigma^* \mid uv \in L \text{ for some string } u \in \Sigma^*\}$. Show that if L is regular, then so is $SUFFIX(L)$. (Hint: construct an NFA for $SUFFIX(L)$.)

PROBLEM 7 (Extra Credit: 2 points)

A queue is similar to a stack, except that pushing and popping happen at *opposite* ends. That is, symbols are pushed onto the top of the queue, and symbols are read and popped off the bottom of the queue. A QA is a nondeterministic automaton just like a PDA, but it has a queue instead of a stack. Find a language that is recognized by a QA but not a PDA. (There is no need to provide a formal definition of a QA.)