

**Harvard University  
Computer Science 121**

**Midterm Solution 15-Oct-98**

The points on the exam total 80. You may use any theorems proven in class or on the homeworks. Good luck! PLEASE NOTE: This examination is two pages. Please take care to do both.

PROBLEM 2 (4+4+4+4+4+4+1 points)

True or false? If true, justify your answer in four sentences or less. If false, provide a counterexample.

- (A)  $P(\{n \in \mathbb{N} : n \leq 2002\})$  is uncountable
- (B)  $\{w : w \text{ is a regular expression for } \{a^n b^m : n + m \leq 2002\}\}$  is a finite language
- (C) The concatenation of a regular language and a nonregular language is necessarily nonregular
- (D)  $(L^+)^+ = L^+ L^+$
- (E)  $\emptyset \subseteq \{\emptyset, a\}$
- (F) If a DFA  $M$  contains a self-loop on some state  $q$ , then  $M$  must accept an infinite language
- (G) Your favorite TF has blonde hair.

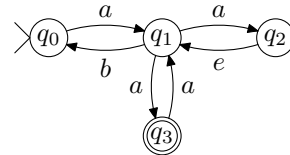
PROBLEM 3 (5+5+6 points)

Are each of the following languages regular or non-regular? Justify your answer by either drawing a DFA/NDFA, giving a regular expression (if you think it's regular), or using closure properties.

- (A)  $\{a^i b^j a^k b^l : i, j, k, l \geq 0\}$
- (B)  $\{w \in \{a, b\}^* : \text{the number of } a\text{'s in } w \text{ times the number of } b\text{'s in } w \text{ is odd}\}$
- (C)  $\{a^n (bc)^n : n \geq 0\}$  [*Hint*: Define a homomorphism and use the closure property you proved on the last problem set.]

PROBLEM 4 (8+4+7 points)

- (A) Consider the adjacent NDFA. Does this NDFA accept  $a$ ?  $aa$ ?  $aaaba$ ?  $aba$ ? [You do not need to show your work.]
- (B) Trace all computations of this NDFA on the string  $aaa$ .



- (C) Consider the relation “Is reachable from in one transition or more” over the set of states of this NDFA. Is this relation reflexive? Symmetric? Transitive? An equivalence relation? Which, if any, of these answers would change if the relation was over sets of states of an *arbitrary* NDFA?

(TURN PAGE OVER - DON'T WORRY, THERE'S MORE!)

PROBLEM 5 (5+7+5 points)

For each of the DFAs drawn below, give both an informal description of and a regular expression for the language it accepts. Do not use the construction described in class: just build the regular expression from your informal description. Your descriptions should be short, and need not be backed by proofs.

