

**Harvard University
Computer Science 121**

Problem Set 5

Due Tuesday October 16, 2012 at 11:59 PM.

Submit your solutions electronically on the course website, located at <http://people.seas.harvard.edu/~salil/cs121/fall12/>. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A, B, and C in separate files named lastname-ps5a.pdf, lastname-ps5b.pdf, and lastname-ps5c.pdf respectively, in the appropriate dropboxes.

Late problem sets may be turned in until Friday October 19, 2012 at 11:59 PM with a 20% penalty.

Problem set by ****ENTER YOUR NAME HERE****

Collaboration Statement: ****FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)****

See syllabus for collaboration policy.

PART A (Graded by Jiapeng)

PROBLEM 1 (10 points)

Draw a PDA that recognizes the language $A = \{a^n b^m : n \leq 2m \leq 4n\}$. Explain briefly how it works.

PROBLEM 2 (4+4+4 points)

Prove or disprove that context-free languages are closed under:

- (A) Reversal.
- (B) Set difference. (The difference of two sets is defined as $L_1 - L_2 := \{x \in L_1 : x \notin L_2\}$.)
- (C) Homomorphism. (See ps2 for a definition of homomorphisms on languages.)

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PART B (Graded by Joe)

PROBLEM 3 (10 points)

Define $\text{Fraternal-Twin}(L) = \{wy : |w| = |y| \text{ and } w, y \in L\}$. Show that if L is regular, then $\text{Fraternal-Twin}(L)$ is context-free.

PROBLEM 4 (4+4+4+4 points)

Are the following languages context-free? Prove your answers.

- (A) $\{a^n b^{n^2} : n \geq 0\}$.
- (B) $\{a^i b^j c^k : i, j, k \geq 0 \text{ and } i + j = k\}$.
- (C) $\{a^n b^n c^i : i \leq n\}$.
- (D) $\{x \in \{a, b\}^* : x \neq ww \text{ for any } w \in \Sigma^*\}$.

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PART C (Graded by Bo)

PROBLEM 5 (5 + 5 + 1 points)

As we know, (nondeterministic) PDAs are much stronger than NFAs. It is natural to ask whether the addition of extra stacks would make PDAs even more powerful.

- (A) Give a formal definition of a (nondeterministic) k -stack PDA and how it computes.
- (B) Show that 2-stack PDAs are more powerful than 1-stack PDAs, meaning that the class of languages recognized by 2-stack PDAs is a strict superset of the class recognized by 1-stack PDAs.
- (C) (Challenge!) Now show that 3-stack PDAs are no more powerful than 2-stack PDAs.