CS 121 Section 6

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October 24, 2013

1 Overview

This week we will discuss Turing machines and the Church-Turing thesis.

1.1 Turing Machines

Formally, a Turing machines is a 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$, where

- Q is the set of states.
- Σ is the input alphabet.
- Γ is the tape alpabet. We need $\Sigma \subset \Gamma$ and we have a special blank symbol $\sqcup \in \Gamma - \Sigma$.
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function.
- $q_0, q_{\text{accept}}, q_{\text{reject}} \in Q$ are the start state, accept state, and reject state respectively

A Turing machine configuration is a string $uqv \in \Gamma^* Q \Gamma^*$ that encodes (i) the state q of M, (ii) the tape contents uv, and (iii) the location of the head within the tape. Note that we ignore trailing blanks so $uqv\sqcup$ and uqv are considered equivalent configurations.

Configurations are updated as follows.

- $uq\sigma v \to u\sigma' q' v$ if $\delta(q, \sigma) = (q', \sigma', R)$.
- $u\tau qv\sigma \to uq'\tau\sigma' v$ if $\delta(q, \sigma) = (q', \sigma', L)$.
- $q\sigma v \to q'\sigma' v$ if $\delta(q, \sigma) = (q', \sigma', L)$.

A computation starts in the configuration $q_0 x$, where x is the input. It accepts if it enters a configuration of the form $uq_{\text{accept}}v$. It rejects if it enters a configuration of the form $uq_{\text{reject}}v$. It is possible that M never accepts and never rejects; if this happens, we say that M does not halt.

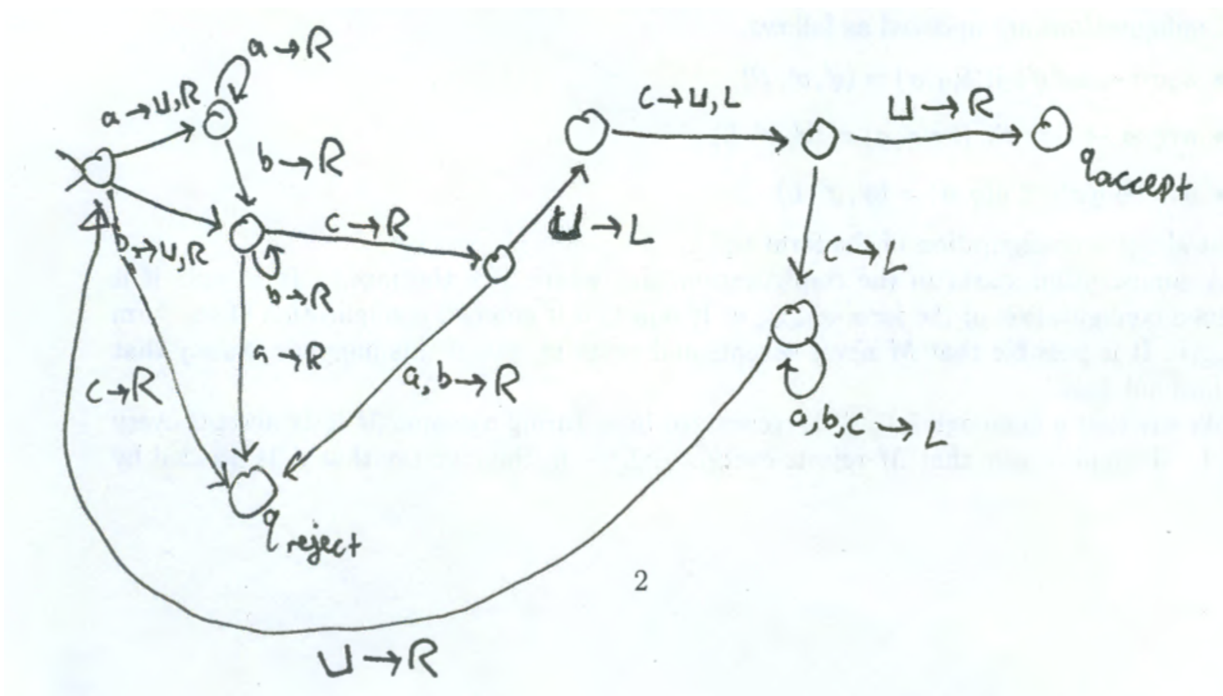
We say that a language $L \subseteq \Sigma^*$ is *recognized* by a Turing machine M if M accepts every $x \in L$. If we also have that M rejects every $x \in \Sigma^* - L$, then we say that L is *decided* by M.

1.2 Church-Turing thesis

The Church-Turing thesis is the assertion that Turing machines capture our intuitive notion of computability. This is not a mathematical statement! We cannot prove it, but we have good reasons to believe it nonetheless. There are many equivalent models of computation. (e.g. General grammars (PS6), multitape TMs, recursive functions, λ-calculus.)

2 Excercises

Exercise 2.1. *Construct a TM that decides the language* $L = \{a^n b^m c^{n+m} : n, m \geq 0\}$.



Exercise 2.2. *Show that the language*

$$L = \{\langle M, w\rangle : M \text{ never moves its head left when running on } w\}$$

is recognizable.

We have to prove that if a TM does not move left in a finite number of steps then it never moves left. So assume the TM has q states and w is a string of length n. After $n-1$ steps, the TM now only reads blanks. After $q+2$ steps right moving steps over blanks, the TM must have at least gone through a state q_i and returned to that state again. But the TM is in exactly the same configuration it was the first time it got to q_i (current state: q_i, input: blank cell, next state: q_i). Hence we are in an infinite loop, but since the TM moved right on the first pass of the loop, it will continue moving right infinitely. Our decider D will count the number of states in w and the length of w. It will then simulate M on w, if in $n+q+1$ steps, M has not moved left, D will accept. Otherwise, reject.

Exercise 2.3. *Imagine a special Turing Machine with a 2-dimensional tape. So instead of the usual linear tape this TM has an infinite upper-right quadrant where the head starts at position* $(0,0)$*. Upon reading each symbol this 2D Tape TM can choose to move left, right, up, or down (but of course cannot move off the edge). The input string* w *will start along the bottom row of the 2D tape, from positions* $(0,0)$ *to* $(n,0)$*. Show that this 2D Tape TM is no more powerful than a standard TM by simulating a 2D Tape TM with a normal one.*

Exercise 2.4. *Show that we can assume that a TM always halts with an empty tape. That is, show how to convert a TM* M *into a TM* M' *with* $L(M) = L(M')$ *where the configuration of* M' *when it accepts or rejects is either* q_{accept} *or* q_{reject} *and not* $uq_{accept}v$ *or* $uq_{reject}v$ *with* $uv \notin \{\sqcup\}^*$.