Midterm Review

October 10, 2011

Important Things

Here are some things you should make sure to study:

- Definition of DFA, NFA, Regular Expressions
- How to convert between DFAs, NFAs, and Regular Expressions
- Closure properties of regular languages
- Countability and Uncountability
- Pumping Lemma
- Definition of CFG

Practice Problems

1. Are the following true or false? Let L_1 and L_2 be arbitrary languages over an alphabet $\Sigma = \{a, b\}$.
   (a) If L_1 is infinite, then $L_1 \circ L_2$ is infinite
   (b) $(L_1^*)^* = L_1^*$
   (c) $L_1^* = (L_1 L_1)^*$
   (d) $(L_1 \circ L_2)^* = L_1^* \circ L_2^*$
   (e) If $L_1 \subset L_2$, then $L_1^* \subset L_2^*$
   (f) If L_1 is regular, then it is context-free.
   (g) If L_1 is finite, then it is regular.
   (h) If L_1 is non-regular, then $\overline{L_1}$ is non-regular.
2. Draw a DFA that recognizes the following languages. Assume $\Sigma = \{a, b\}$
   (a) $\{w \mid |w| = 5\}$
   (b) All strings except the empty string
3. Draw an NFA the recognizes the language a^*b^*aba
4. Are the following languages regular or not?

(a) $\{x = y + z \mid x, y, z$ are binary integers, and x is the sum of y and $z\}$ (the alphabet here is $\{0, 1, =, +\}$).

(b) $\{w \mid w$ is the binary representation of a number greater than $3\}$

5. Give a context free grammar for the following languages:

   (a) The set of strings over $\{a, b\}$ with more a's than b's.

   (b) $\{a^i b^j c^k \mid i = j$ or $j = k$ and $i, j, k \geq 0\}$.

6. Sometimes it is easier to prove properties of regular languages by basing the argument on DFAs. Other times it is easier to work with NFAs or Regular Expressions. Each of the following are easier to prove using one of these three representations (in the humble opinion of one of your TFs). Prove the following statements.

   (a) If L_1 is a regular language, then $\overline{L_1}$ is regular.

   (b) For all languages L, define $DROPOUT(L) = \{w \mid w$ is some string from L with exactly one character deleted$\}$. Prove that if L is regular, then $DROPOUT(L)$ is regular.