CS 121, Section 2

Week of September 16, 2013

1 Concept Review

1.1 Overview

In the past weeks, we have examined the *finite automaton*, a simple computational model with limited memory. We proved that DFAs, NFAs, and regular expressions are equal in computing power and recognize the *regular languages*. We also showed that the regular langages are closed under *union, concatenation, Kleene Star, intersection, difference, complement, and reversal.* We then used a counting argument to show that there are indeed languages which are non-regular.

This week in section we will become a little more comfortable with these topics by working with regular expressions, making arguments about countability, and exploring some more closure properties of regular languages.

1.2 Cardinalities

We classify the cardinality of a set S as follows.

- Finite, if there is a bijection between S and $\{1, 2, \cdots, n\}$ for some $n \geq 0$.
- Coutably infinite, if there is a bijection between S and $\mathbb{N}$.
- Countable, if it is finite or countably infinite.
- Uncountable, otherwise.

Examples include the following.

- Finite: Σ (alphabet), states of a DFA, students in CS121, finite unions of finite sets.
- Countably infinite: Σ^* (strings), $\mathbb{Z}$, DFAs, countable unions of countably infinite sets.
- Uncountable: $\mathcal{P}(\mathbb{N})$, set of all languages.

Since there are only countably many regular languages and uncountably many languages, 'most' languages are non-regular.

2 Exercises

Exercise 2.1. *Describe in plain English the language represented by the following regular expressions.*

(a) $a^* \cup b^*$

(b) $(aaa)^*$

(c) $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$

Exercise 2.2. *Using the procedure outlined in class, convert the regular expression* $(((aa)^*(bb)) \cup ab)^*$ *to an equivalent NFA.*

Exercise 2.3. *Let* L *be a language over the alphabet* $\Sigma = \{a, b\}$. *Define* $\mathsf{PigLatin}(L) = \{w\sigma : \sigma \in \Sigma, w \in \Sigma^*, \sigma w \in L\}$. *Informally,* $\mathsf{PigLatin}(L)$ *is the language containing all strings in* L *except that each string has had its first character moved to its end. (For example,* $\mathsf{PigLatin}(\{abc, a, aab\}) = \{bca, a, aba\}$*.)*

Show that if L *is regular, then* $\mathsf{PigLatin}(L)$ *is regular. Specifically, given a DFA for* L*, show how to construct an NFA for* $\mathsf{PigLatin}(L)$*. (Your proof for this problem should involve finite automata and not regular expressions.)*

Exercise 2.4. *Prove or disprove the following statements about regular expressions:*

1. $L((R \cup S)^*) = L(R^* \cup S^*)$
2. $L((RS \cup R)^* R) = L(R(SR \cup R)^*)$
3. $L((RS \cup R)^* RS) = L((RR^* S)^*)$

Exercise 2.5. *Are the following sets finite (if so, how large), countably infinite, or uncountably infinite? Justify your answer.*

1. *The set of all infinite binary sequences* $\{0, 1\}^{\mathbb{N}}$
2. *The set of real numbers* $\mathbb{R}$.
3. *The set of rational numbers* $\mathbb{Q}$.
4. *The set of all English words.*
5. *The set of all English sentences.*