Section 3 Handout

CS 121

September 26, 2013

Today's Topics

- Regular and Non-regular Languages
- Context-Free Grammars

1 Regular and Nonregular Languages

There are countably many regular expressions over a language, but there are uncountably many languages–so some of these languages *must not* be regular! But how do we find an explicitly non-regular language? We have two techniques: the pumping lemma and the closure properties of regular languages. You can use either of these techniques to prove (by contradiction) that a language is non-regular.

Pumping Lemma for regular languages:
If L is a regular language, then there exists a constant $p > 0$ such that for any string $w \in L$ with $|w| > p$, there exist strings x, y, $z \in \Sigma^*$, such that $w = xyz$, $|xy| \leq p$, $y \neq \epsilon$, and $xy^n z \in L$ for all $n \geq 0$.

Closure Properties:
Recall from lecture (and from last week's section) that regular languages are closed under *union, concatenation, Kleene Star, intersection, difference, complement, reversal.*

Exercise 1.1. *Which of the following are necessarily regular?*

- *A finite language.*
- *A union of finitely many regular languages.*
- $\{x : x \in L_1$ *and* $x \notin L_2\}$ *where* L_1 *and* L_2 *are regular.*
- *A superset of a regular language.*

Exercise 1.2. *Show that* $L = \{a^i b^j : 0 \le i < j\}$ *is non-regular using the pumping lemma.*

Exercise 1.3. *Let* $L = \{ww | w \in \Sigma^*\}$*. Show that* L *is non-regular using the pumping lemma.*

Exercise 1.4. *Let* $L = \{w : w$ *has more instances of substring* aa *than of substring* $bb\}$ *Show that* L *is nonregular.*

Exercise 1.5. *Show that* $L = \{b^n c^{2^k} : n \ge 1, k \ge 1\}$ *is non-regular.*

2 Context-Free Languages

Context-Free Grammars: A context-free grammar G is a four-tuple, defined as follows: $G = (V, \Sigma, R, S)$, where V (the set of variables) is an alphabet, Σ (the set of terminals) is a set disjoint from V, R is a finite set of rules, with each rule being a variable and a string of variables and terminals, and S (the start symbol) is an element of V.

Exercise 2.1. *Give a context-free grammar for* $L = \{w :$ *w is an even-length palindrome* $\}$

Exercise 2.2. *(a) Give a context-free grammar for* $L = \{w : w$ *has three more a's than b's*$\}$ *over the alphabet* $\Sigma = \{a, b\}$

(a) Draw a parse tree for the string $baabaaa \in L$.

Exercise 2.3. *Let* $L = \{wy : w, y \in L(a^* \cup b^*)$ *and* $|w| = |y|\}$*. Is* L *regular? Is* L *context-free?*