CS 121 Section 5

Harvard University

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Overview

This week we will focus on reviewing the core concepts involved with PDAs, Pumping lemma for CFL, and closure properties of context-free language.

1 Concept Review

1.1 PDA

Intuitively a Pushdown Automata (PDA) represents a NFA with an additional (unbounded size) stack. The addition of the stack makes PDAs much powerful than NFA. For example, the language $L = \{a^n b^n : n \in \mathbb{N}\}$ is recognized by a PDA, but not by any NFA. Intuitively, PDAs the stack gives PDAs the ability to perform simple counting tasks.

1.2 PDAs v.s. CFGs

PDAs and Context-Free Grammars are equivalent in power.

Theorem 1.1. *The CFLs are the languages accepted by PDAs.*

1.3 Pumping lemma for Context-Free Language

Lemma 1.1. *If L is context-free, then there is a number p such that any $s \in L$ of length at least p can be divided into $s = uvxyz$, where*

- $v \neq \epsilon$ *or* $y \neq \epsilon$,
- $|vxy \leq p|$ *,and*
- $uv^i xy^i z \in L$ *for every* $i \geq 0$.

1.4 Closure properties of Context-Free Language

The Context-Free Languages are closed under:

- Union
- Concatenation
- Kleene $*$
- Intersection with a regular set

The Context-Free Languages are **not** closed under:

- Complement
- Intersection

2 Exercises

Exercise 2.1. *Explain and justify the following statement: "Almost all languages are not context-free."*

Exercise 2.2. *Show that* $L = \{a^n b^{2n}\}$ *is context-free by giving a PDA that accepts it. Draw the state diagram and write the 6-tuple* $(Q, \Sigma, \Gamma, \delta, q_0, F)$

Exercise 2.3. *Determine, with proof, whether or not each of the following languages is context-free.*

1. $\{ww : w \in \Sigma^*\}$
2. $L = \{w \text{ is not of the form } a^n b^n\}$
3. $\{w \in \{(,)\}^* : w \text{ is not properly parenthesized}\}$
4. $L = \{a^i b^j c^k | 0 \leq i \leq j \leq k\}$

Exercise 2.4. *DPDAs*

1. *Give a formal definition of a deterministic pushdown automaton (DPDA). (Note, there are a few different ways of doing this. Some defintions are equivalent, but some are not. However, any definition that eliminates nondeterminism from PDAs will yield less powerful machines.)*
2. *Now show that the DPDAs defined in part 1 are weaker than PDAs.*